Crossover from Fermi to Non-Fermi Liquid in Two-Dimensional Interacting Fermions

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Self-energy at zero temperature is investigated up to the third-order of interaction using one-patch model in two dimensions, whose interaction process corresponds to g_4-process of g-ology model in one dimension. The self-energy $\Sigma^R(\boldsymbol{k},\epsilon)$ diverges at $\epsilon = \xi_k$, and the contribution from the particle-hole process in third-order self-energy diagrams has a stronger divergence compared to the one from the particle-particle process. This implies that the loop-cancellation in the forward scattering is insufficient due to the effect of the warping of the Fermi surface. The strong energy dependence of the self-energy in the vicinity of $\epsilon = \xi_k$ implies the existence of the crossover from Fermi to non-Fermi liquid behavior as the momentum becomes away from the Fermi momentum, and this crossover is enhanced as interaction becomes stronger.

KEYWORDS: two dimensional electron gas, forward scattering, crossover from Fermi to non-Fermi liquid, loop-cancellation

The low-energy excitation of the interacting fermions with short-range force is established as Tomonaga-Luttinger (TL) liquid in one dimension and Fermi liquid in three dimensions, while as for two dimensions it is still in controversy. In case of one dimension, forward scattering processes, which are denoted g_2- and g_4-process in g-ology model, leads to TL liquid.[1)] The main features of TL liquid are the following two things: vanishing jump of momentum distribution at Fermi momentum and spin-charge separation. The g_2-process is related to the former, and the g_4-process to the latter in the following sense. Taking account of only $g_{2\parallel}$- or $g_{2\perp}$-process, the velocity of spin and charge excitations, v_ρ and v_σ, respectively, become different, which indicates the existence of spin-charge separation and results in the two-peak structure of spectral-weight.[2,3)] On the other hand, if we consider only $g_{4\perp}$-process (here $g_{4\parallel}$-process related term cancels if we neglect momentum dependence of the coupling constant), the parameters K_ρ and K_σ, which equal to 1 for free fermions and characterize anomalous power-laws of various correlation functions, deviate from 1. This also leads to vanishing jump of momentum distribution at Fermi surface.

In case of two dimensions, it was suggested that anomalous behavior of forward scattering phase-shift leads to non-Fermi liquid even at weak-coupling quite similarly to TL liquid.[4,5)] But in this stage, there is no theory which confirms this possibility. There is also no signal of non-Fermi liquid state from many-body perturbation approach in two dimensions.[6–17)] In the following we investigate this possibility from perturbation theory in detail.

To start with, we consider the following correspondence between the model in one and two dimensions. We obtain low-energy effective theories by integrating out degrees of freedom of electrons far from Fermi points (or surface), which is so-called the elimination of fast modes.[10,18)] In one dimension, the low-energy effective theory is g-ology model, and there are two branches corresponding to two Fermi points. In case of two dimensions, the low-energy effective theory has only degrees of freedom of electrons in a thin shell with thickness Λ around the Fermi surface. Since momenta of electrons are allowed only within the thin shell, interaction processes are extremely restricted; only three kinds of processes shown in fig. 1, i.e., forward, exchange and Cooper scatterings, are allowed (here we neglect Umklapp process).[10,18)] Dividing the thin shell around the Fermi surface to many small patches of the size $\Lambda \times \Lambda$, we obtain the following low-energy effective action for zero temperature;

$$\mathcal{S} \equiv \mathcal{S}_0 + \mathcal{S}_{forward} + \mathcal{S}_{exchange} + \mathcal{S}_{Cooper} \tag{1a}$$

$$\mathcal{S}_0 = \sum_\sigma \int_{k\epsilon} Z_k^{-1}(i\epsilon - \xi_k) c^\dagger_{k,\sigma} c_{k,\sigma} \tag{1b}$$

$$\mathcal{S}_{forward} = -\frac{1}{2}\sum_{ij}\sum_{\sigma\sigma'}\int_{k\epsilon}\int_{k'\epsilon'}\int_{q\omega}$$

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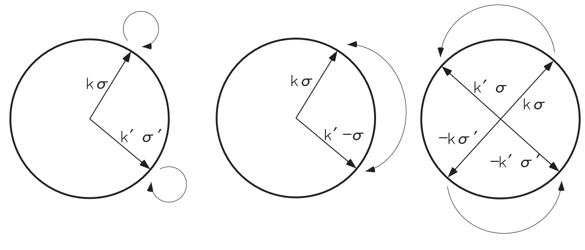

(a) (b) (c)

Fig. 1. Three kinds of interaction processes. (a)Forward scattering ($k\sigma, k'\sigma' \to k\sigma, k'\sigma'$). (b)Exchange scattering ($k\sigma, k'-\sigma \to k'-\sigma, k\sigma$). (c)Cooper scattering ($k\sigma, -k\sigma' \to k'\sigma, -k'\sigma'$).

$$g_{F}{}^{\sigma\sigma'}_{kk'}(q) c^{\dagger}_{k+q,\sigma} c^{\dagger}_{k'-q,\sigma'} c_{k',\sigma'} c_{k,\sigma} \tag{1c}$$

$$\mathcal{S}_{exchange} = -\frac{1}{2}\sum_{i\neq j}\sum_{\sigma}\int_{k\epsilon}\int_{k'\epsilon'}\int_{q\omega} g_{E}{}^{\sigma -\sigma}_{kk'}(q) c^{\dagger}_{k+q,\sigma} c^{\dagger}_{k'-q,-\sigma} c_{k,-\sigma} c_{k',\sigma} \tag{1d}$$

$$\mathcal{S}_{Cooper} = -\frac{1}{2}\sum_{i\neq j}\sum_{\sigma\sigma'}\int_{k\epsilon}\int_{k'\epsilon'}\int_{q\omega} g_{C}{}^{\sigma\sigma'}_{kk'}(q) c^{\dagger}_{k,\sigma} c^{\dagger}_{-k-q,\sigma'} c_{-k'+q,\sigma'} . c_{k',\sigma} \tag{1e}$$

where

$$\int_{k\epsilon} \equiv \int \frac{d^2 k}{(2\pi)^2}\int_{-\infty}^{\infty}\frac{d\epsilon}{2\pi}, \tag{2}$$

and $c^{\dagger}_{k,\sigma}$ and $c_{k,\sigma}$ are Grassmann variables for fermion with momentum and energy $k = (\boldsymbol{k}, i\epsilon)$ and spin σ, and g_F, g_E and g_C are coupling constants for forward, exchange and Cooper processes. Denoting patches of the size $\Lambda\times\Lambda$ as Λ_i (i is an index of patch), the integration for momenta $\boldsymbol{k}$, $\boldsymbol{k'}$ and $\boldsymbol{q}$ in eqs. (1c), (1d) and (1e) are performed in the region where $k, k+q \in \Lambda_i$ and $k', k'-q \in \Lambda_j$ are satisfied. $\mathcal{S}_{Cooper}$ is a term related to Cooper instability in case of attractive interaction. Regarding patches in two dimensions as analogs of branches in one dimension, we can make correspondence from g-ology model to the model given by the action of the form eq. (1). Namely, $\mathcal{S}_{forward}$ corresponds to g_2- and g_4-terms, and $\mathcal{S}_{exchange}$ to g_1-term in g-ology model. As for $\mathcal{S}_{forward}$, the case $i = j$ and $i \neq j$ correspond to g_2- and g_4-terms, respectively.[10,18]

In one dimension, forward scatterings, i.e., g_2- and g_4-processes, lead to TL liquid, and the question is what character the low-energy excitations have in the presence of the term $\mathcal{S}_{forward}$ in eq. (1) in two dimensions. In the following, we consider the simplest case, in which there exists only $i = j$ term in $\mathcal{S}_{forward}$, and calculate the self-energy up to the third order of interaction. In one dimension, this process leads to the spin-charge separation. Following the procedure shown graphically in fig. 2, we introduce the model described by an action of the form $\mathcal{S} = \mathcal{S}_0 + \mathcal{S}_I$, where

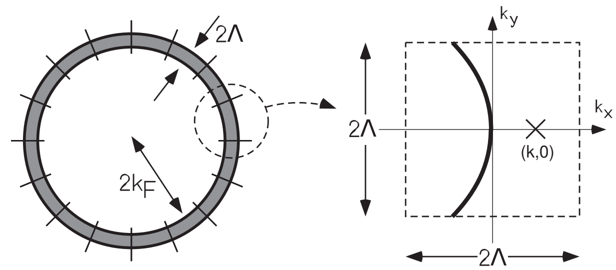

Fig. 2. Taking account of only the process corresponding to g_4-process in one dimension, we make a one-patch model.

$$\mathcal{S}_0 = \sum_{\sigma}\int_{k\epsilon} Z^{-1}(i\epsilon - \xi_k) c^{\dagger}_{k,\sigma} c_{k,\sigma} \tag{3}$$

and

$$\mathcal{S}_I = -\frac{U}{2}\sum_{\sigma}\int_{k\epsilon}\int_{k'\epsilon'}\int_{q\omega} c^{\dagger}_{k+q,\sigma} c^{\dagger}_{k'-q,-\sigma} c_{k',-\sigma} c_{k,\sigma}. \tag{4}$$

Assuming $g_{F}{}^{\sigma -\sigma}_{kk'}(q)$ is an analytic function in the vicinity of $k = k'$ and $q = 0$, and neglecting k, k' and q-dependences of $g_{F}{}^{\sigma -\sigma}_{kk'}(q)$ in a patch, we have replaced $g_{F}{}^{\sigma -\sigma}_{kk'}(q)$ to a constant U in eq. (4). The renormalization factor Z_k is also replaced to the constant Z in eq. (3). The origin of momentum and k_x and k_y-axes are taken as shown in fig. 2, and momentum cut-offs are introduced as $|k_x|, |k_y| < \Lambda$. We approximate the energy dispersion as[19)]

$$\xi_k = v k_x + \frac{A}{2}k_y^2. \tag{5}$$

Furthermore, we define cut-off energies $\tilde{\epsilon}_x$ and $\tilde{\epsilon}_y$ and the constant $\tilde{U}$ for later convenience as

$$\tilde{\epsilon}_x \equiv v\Lambda,\ \tilde{\epsilon}_y \equiv A\Lambda^2,\ \tilde{U} \equiv U\Lambda^2 \tag{6}$$

Using this model, we evaluate the contributions of the diagram shown in figs. 3(a) and 3(b), which we denote $\Sigma^R_{pp}(\boldsymbol{k}, \epsilon)$ and $\Sigma^R_{ph}(\boldsymbol{k}, \epsilon)$, respectively, where $\boldsymbol{k} = (k, 0)$.

Firstly, we consider the contribution of the diagram in fig. 3(a), $\Sigma^R_{pp}(\boldsymbol{k}, \epsilon)$. $\Sigma_{pp}(\boldsymbol{k}, \epsilon + i\delta)$ is expressed as

$$\Sigma^R_{pp}(\boldsymbol{k}, \epsilon) = U^3\int_{q\omega}\Big[\mathrm{sgn}(\omega)\,\mathrm{Im}\,\big[K^R(\boldsymbol{q}, \omega)\big]^2 G^A_{q-k}(\omega - \epsilon) + \mathrm{sgn}(\omega - \epsilon)\,\big[K^R(\boldsymbol{q}, \omega)\big]^2\,\mathrm{Im}\,G^R_{q-k}(\omega - \epsilon)\Big]. \tag{7}$$

Here $K^R(\boldsymbol{q}, \omega)$ is a particle-particle correlation function

defined as

$$K^R(\boldsymbol{q},\omega) \equiv \int_{kx} \mathrm{sgn}(x)\, G^R_{q-k}(\omega - x)\, \mathrm{Im}\, G^R_k(x), \tag{8}$$

which is expressed approximately for $|\omega|, |vq_x|, |Aq_y^2| \ll \tilde\epsilon_x, \tilde\epsilon_y$ as

$$K^R(\boldsymbol{q},\omega) \simeq \begin{cases} K_0 + \dfrac{iZ^2\omega}{4\pi v A^{1/2}(\omega - vq_x - Aq_y^2/4)^{1/2}} \\ \qquad\qquad (\omega - vq_x - Aq_y^2/4 > 0) \\ K_0 + \dfrac{Z^2\omega}{4\pi v A^{1/2}(-\omega + vq_x + Aq_y^2/4)^{1/2}} \\ \qquad\qquad (\omega - vq_x - Aq_y^2/4 < 0), \end{cases} \tag{9}$$

where

$$K_0 \equiv \lim_{q\to 2k_F} \lim_{w\to 0} K^R(\boldsymbol{q}, w) = \frac{Z^2\Lambda}{2\pi^2 v}. \tag{10}$$

Here the sequence of the limiting procedure is important reflecting the singularity of the particle-particle correlation in the vicinity of $\boldsymbol{q} = 2\boldsymbol{k}_F$. We extract the term which contains singular part of self-energy $\Sigma^R_{pp}(\boldsymbol{k},\epsilon)$ in k and ϵ, which is denoted as $\Sigma^{R\,\prime}_{pp}(\boldsymbol{k},\epsilon)$ and defined as

$$\Sigma^{R\,\prime}_{pp}(\boldsymbol{k},\epsilon) \equiv -U^3 \int \frac{d^2q}{(2\pi)^2} \int_0^\epsilon \frac{d\omega}{\pi} \left[K^R(\boldsymbol{q},\omega)\right]^2 \mathrm{Im}\, G^R_{q-k}(\omega - \epsilon). \tag{11}$$

The analytic part of the self-energy, i.e., $\Sigma^R_{pp}(\boldsymbol{k},\epsilon) - \Sigma^{R\,\prime}_{pp}(\boldsymbol{k},\epsilon)$, is considered to be related to various renormalizations, and whose effect can be absorbed into the renormalizations of the constants v, A and Z. Substituting eq. (9) to eq. (11), we obtain

$$\Sigma^{R\,\prime}_{pp}(\boldsymbol{k},\epsilon) = \begin{cases} c_1\epsilon + \dfrac{iZ^5\tilde U^3 \tilde K_0}{8\pi^3 \tilde\epsilon_x^2 \tilde\epsilon_y}\epsilon^2 \log\dfrac{\tilde\epsilon_y}{|\epsilon - vk|} \\ \qquad -\dfrac{Z^5\tilde U^3}{96\pi^3 \tilde\epsilon_x^3 \tilde\epsilon_y^{3/2}} \dfrac{\epsilon^3}{(\epsilon - vk)^{1/2}} \qquad (\epsilon > vk) \\ c_1\epsilon + \dfrac{iZ^5\tilde U^3 \tilde K_0}{8\pi^3 \tilde\epsilon_x^2 \tilde\epsilon_y}\epsilon^2 \log\dfrac{\tilde\epsilon_y}{|\epsilon - vk|} \\ \qquad +\dfrac{iZ^5\tilde U^3}{96\pi^3 \tilde\epsilon_x^3 \tilde\epsilon_y^{3/2}} \dfrac{\epsilon^3}{(vk - \epsilon)^{1/2}} \qquad (\epsilon < vk), \end{cases} \tag{12}$$

where

$$c_1 \equiv -\frac{U^3}{4\pi^3} \int d^2q \left[K^R(\boldsymbol{q},\omega)\right]^2 \mathrm{Im}\, G^R_q(\omega)\bigg|_{\omega=0}, \tag{13}$$

and $\tilde K_0 = K_0\Lambda^{-2}$. The effect of the first term in eq. (12), $c_1\epsilon$, can be also absorbed into the renormalizations of the constants v, A and Z.

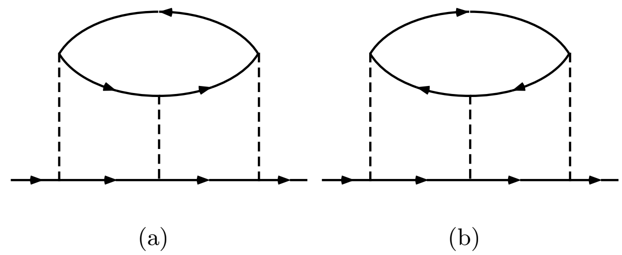

Fig. 3. Third order self-energy diagrams, which are assumed to cancel out in the framework of the loop-cancellation, but do not cancel in the present two-dimensional model. (a) Particle-particle process. (b) Particle-hole process.

Secondly, we consider the contribution of the diagram fig. 3(b), which is denoted as $\Sigma^R_{ph}(\boldsymbol{k},\epsilon)$ and expressed as

$$\Sigma^R_{ph}(\boldsymbol{k},\epsilon) = U^3 \int_{q\omega} \Big[\mathrm{sgn}(\omega)\, \mathrm{Im}\, \left[\chi^R(\boldsymbol{q},\omega)\right]^2 G^R_{q+k}(\omega+\epsilon) + \mathrm{sgn}(\omega+\epsilon)\, \left[\chi^A(\boldsymbol{q},\omega)\right]^2 \mathrm{Im}\, G^R_{q+k}(\omega+\epsilon)\Big]. \tag{14}$$

Here $\chi^R(\boldsymbol{q},\omega)$ is the particle-hole correlation function defined as

$$\chi^R(\boldsymbol{q},\omega) = \int_{kx} \big[\mathrm{sgn}(x)\, G^R_{q+k}(\omega+x)\, \mathrm{Im}\, G^R_k(x) + \mathrm{sgn}(\omega+x)\, G^A_k(x)\, \mathrm{Im}\, G^R_{q+k}(\omega+x)\big]. \tag{15}$$

Substituting the energy dispersion given by eq. (5), we obtain the expressions for $\chi^R(\boldsymbol{q},\omega)$ in case $|\omega|, |vq_x|, |Aq_y^2| \ll \tilde\epsilon_x, \tilde\epsilon_y$ as

$$\chi^R(\boldsymbol{q},\omega) = \begin{cases} \chi_0 - \dfrac{Z^2\omega}{4\pi^2 vAq_y} \log\left(\dfrac{A\Lambda q_y - \omega + vq_x}{-A\Lambda q_y - \omega + vq_x}\right) \\ \qquad\qquad (|\omega - vq_x| > A\Lambda|q_y|) \\ \chi_0 - \dfrac{Z^2\omega}{4\pi^2 vAq_y} \left[\log\left(\dfrac{A\Lambda q_y - \omega + vq_x}{A\Lambda q_y + \omega - vq_x}\right) + i\pi\, \mathrm{sgn}[q_y]\right] \\ \qquad\qquad (|\omega - vq_x| < A\Lambda|q_y|). \end{cases} \tag{16}$$

Here χ_0 is defined as

$$\chi_0 \equiv \lim_{q\to 0} \lim_{\omega\to 0} \chi^R(\boldsymbol{q},\omega) = -\frac{Z^2\Lambda}{2\pi^2 v}, \tag{17}$$

which is proportional to the density of state at the Fermi energy. In the same way as in case of the evaluation of $\Sigma^R_{pp}(\boldsymbol{k},\epsilon)$, we obtain the singular part of $\Sigma^R_{ph}(\boldsymbol{k},\epsilon)$ by

$\Sigma^{R\,\prime}_{ph}(\boldsymbol{k},\epsilon)$

$$= U^3\int\frac{d^2q}{(2\pi)^2}\int_{-\epsilon}^{0}\frac{d\omega}{\pi}\,\left[\chi^A(\boldsymbol{q},\omega)\right]^2\,\mathrm{Im}\,G^R_{q+k}(\omega+\epsilon), \quad (18)$$

which is evaluated from eqs. (16) and (18) in case of $\epsilon > vk$, for example, as follows;

$$\begin{aligned}
&{\Sigma^R_{ph}}'(\boldsymbol{k},\epsilon) \simeq c_2\epsilon \\
&-\frac{U^3Z^5\chi_0\epsilon^2}{16\pi^4v^2A}\left\{\int_0^{\frac{\epsilon-vk}{A\Lambda}}dq_y\frac{1}{q_y}\left[\log\frac{A\Lambda q_y+(\epsilon-vk)}{-A\Lambda q_y+(\epsilon-vk)}\right]\right.\\
&\qquad\left.+\int_{\frac{\epsilon-vk}{A\Lambda}}^{2\Lambda}dq_y\frac{1}{q_y}\left[\log\frac{A\Lambda q_y+(\epsilon-vk)}{A\Lambda q_y-(\epsilon-vk)}-i\pi\right]\right\}\\
&-\frac{U^3Z^5\epsilon^3}{96\pi^6v^3A^2}\left\{\int_0^{\frac{\epsilon-vk}{A\Lambda}}dq_y\frac{1}{q_y^2}\left[\log\frac{A\Lambda q_y+(\epsilon-vk)}{-A\Lambda q_y+(\epsilon-vk)}\right]^2\right.\\
&\qquad\left.+\int_{\frac{\epsilon-vk}{A\Lambda}}^{2\Lambda}dq_y\frac{1}{q_y^2}\left[\log\frac{A\Lambda q_y+(\epsilon-vk)}{A\Lambda q_y-(\epsilon-vk)}-i\pi\right]^2\right\}\\
&\simeq c_2\epsilon+\frac{iU^3Z^5\chi_0}{8\pi^3v^2A}\epsilon^2\log\frac{A\Lambda^2}{|\epsilon-vk|}\\
&\qquad\qquad+\frac{i(\log 2)U^3Z^5\Lambda}{24\pi^5v^3A}\frac{\epsilon^3}{\epsilon-vk}, \quad (19)
\end{aligned}$$

where

$$c_2 \equiv \frac{U^3}{4\pi^3}\int d^2q\left[\chi^R(\boldsymbol{q},\omega)\right]^2\,\mathrm{Im}\,G^R_q(\omega)\bigg|_{\omega=0}. \quad (20)$$

Evaluating ${\Sigma^R_{ph}}'(\boldsymbol{k},\epsilon)$ for $\epsilon < vk$ in the same way, we obtain the final result as

$$\begin{aligned}{\Sigma^R_{ph}}'(\boldsymbol{k},\epsilon) \simeq c_2\epsilon &+ \frac{iZ^5\tilde{U}^3\tilde{\chi}_0}{8\pi^3\tilde{\epsilon}_x^2\tilde{\epsilon}_y}\epsilon^2\log\frac{\tilde{\epsilon}_y}{|\epsilon-vk|}\\
&+\frac{i(\log 2)Z^5\tilde{U}^3}{24\pi^5\tilde{\epsilon}_x^3\tilde{\epsilon}_y}\frac{\epsilon^3}{\epsilon-vk}, \quad (21)\end{aligned}$$

where $\tilde{\chi}_0 = \chi_0\Lambda^{-2}$.

As for second-order term, we can obtain the singular part in the same way as follows;

$${\Sigma^R_{2nd}}'(\boldsymbol{k},\epsilon) \simeq c_3\epsilon - \frac{iU^2Z^3\epsilon^2}{16\pi^3v^2A}\log\frac{A\Lambda^2}{|\epsilon-vk|}, \quad (22)$$

where

$$c_3 \equiv \frac{U^2}{4\pi^3}\int d^2qK^R(\boldsymbol{q},\omega)\mathrm{Im}\,G^R_q(\omega)\bigg|_{\omega=0}. \quad (23)$$

From eqs. (12) and (21), we have to notice that there exists a region where $\mathrm{Im}\,\Sigma^R(k,\epsilon)$ has a positive value, although $\mathrm{Im}\,\Sigma^R(k,\epsilon)$ has to be negative-definite. This is due to the reason that we have considered only up to the third order. Eqs. (12), (21) and (22) implies the possibility that the spectral-weight $\pi^{-1}|\mathrm{Im}\,G^R(k,\epsilon)|$ has a two-peak structure in the vicinity of $\epsilon = \xi_k$ due to the divergence of the self-energy at $\epsilon = \xi_k$. The singular part of the self-energy is relevant only in the vicinity of $\epsilon = \xi_k$, and we can define the width $\Delta(k)$ of the structure of the spectral-weight in this vicinity as $\Delta(k) = |\Sigma^R(k, vk + \Delta(k))|$. Evaluating $\Delta(k)$ for ${\Sigma^R_{pp}}'$, ${\Sigma^R_{ph}}'$ and ${\Sigma^R_{2nd}}'$, which we denote $\Delta_{pp}(k)$, $\Delta_{ph}(k)$ and $\Delta_{2nd}(k)$, respectively, we obtain $\Delta_{pp}(k) \simeq (Z^{10/3}\tilde{U}^2\tilde{\epsilon}_x^{-2}\tilde{\epsilon}_y^{-1})|vk|^2$, $\Delta_{ph}(k) \simeq (Z^{5/2}\tilde{U}^{3/2}\tilde{\epsilon}_x^{-3/2}\tilde{\epsilon}_y^{-1/2})|vk|^{3/2}$ and $\Delta_{2nd}(k) \simeq (Z^3\tilde{U}^2\tilde{\epsilon}_x^{-2}\tilde{\epsilon}_y^{-1})|vk|^2\log(\tilde{\epsilon}_y/|vk|)$. Noticing the exponents of $|vk|$ in the expressions for $\Delta(k)$, $\Delta_{ph}(k)$ is dominant for small $|k|$ (here the origin of k is taken on the Fermi surface) reflecting the fact that ${\Sigma^R_{ph}}'$ has a stronger divergence at $\epsilon = vk$ than ${\Sigma^R_{pp}}'$, which indicates that the particle-hole process (fig. 3(b)) has a tendency to enhance the splitting of the spectral-weight in the vicinity of $\epsilon = \xi_k$. We can also see $\Delta(k) \ll |\xi_k|$ for small $|\xi_k|$, which indicates that the quasi-particle picture is valid for sufficiently low-energy. $\Delta(k)$ grows as $|\xi_k|$ becomes larger, which leads to the crossover from Fermi to non-Fermi liquid behavior as the momentum k becomes away from the Fermi momentum. This crossover-energy Δ^c can be defined as $\Delta(k)$ which satisfies $\Delta(k) = |\xi_k|$. Above expressions for $\Delta(k)$ is not applicable for $|vk| \gg \tilde{\epsilon}_x, \tilde{\epsilon}_y$, however, we can see the tendency of the interaction dependence of this crossover energy by assuming above expressions for $\Delta(k)$ for $|\xi_k| \gg \tilde{\epsilon}_x, \tilde{\epsilon}_y$. The explicit expression for $\Delta(k)$ obtained by this way is not valid, but can be considered to reflect the speed of growth of the splitting of the spectral-weight as $|\xi_k|$ becomes larger. Denoting Δ^c evaluated from $\Delta_{pp}(k)$, $\Delta_{ph}(k)$ and $\Delta_{2nd}(k)$ as Δ^c_{pp}, Δ^c_{ph} and Δ^c_{2nd}, we obtain $\Delta^c_{pp} \simeq Z^{-10/3}\tilde{U}^{-2}\tilde{\epsilon}_x^2\tilde{\epsilon}_y$, $\Delta^c_{ph} \simeq Z^{-5}\tilde{U}^{-3}\tilde{\epsilon}_x^3\tilde{\epsilon}_y$ and $\Delta^c_{2nd} \simeq Z^{-3}\tilde{U}^{-2}\tilde{\epsilon}_x^2\tilde{\epsilon}_y\log(Z^{3/2}U/\tilde{\epsilon}_x)$, which indicates $\Delta^c \gg \tilde{\epsilon}_x, \tilde{\epsilon}_y$ for small U. Although this represents that the crossover arises only at higher energy compared to band-width $\tilde{\epsilon}_x$ and $\tilde{\epsilon}_y$ in weak-coupling case, we can see a tendency that the crossover-energy decreases as the interaction U becomes stronger. From above consideration within the scope of weak-coupling, a possibility is expected that the crossover energy becomes smaller than band-width for strong-coupling. This possibility is interesting in relation to the non-Fermi liquid behavior of the High-T_c cuprates in low-doping region.

Furthermore, we can consider above results from the viewpoint of loop-cancellation;[10] the third-order diagrams in fig. 3 are assumed to cancel out in the framework of loop-cancellation. The loop-cancellation is exact for TL model, while in case of higher dimensions, the cancellation is not complete even for forward scattering process. Metzner et al. made an elementary estimation by power counting about the residual term of the cancellation for loop-diagrams,[20] while the contribution of this residual term of loop-diagrams to the self-energy is not precisely estimated. From eqs. (12) and (21), we can see that the singularities of the contributions from

the two third-order diagrams do not cancel. This implies that the loop-cancellation is insufficient, and the residual contribution is relevant in the vicinity of $\epsilon = vk$. This can be considered as follows; although for completely flat Fermi surface, the exact loop-cancellation holds even in two-dimension, there is a warping of the Fermi surface characterized by the energy scale $\tilde{\epsilon}_y$ in general. In case of $|\epsilon|, |vk| \gg \tilde{\epsilon}_y$, the loop-cancellation is a good approximation, and the self-energy can be expanded by small parameters $\tilde{\epsilon}_y/|\epsilon|, |vk|$. If we consider the low-energy limit, however, the effect of warping becomes crucially important; $\tilde{\epsilon}_y/|\epsilon|, |vk|$ is no more a small parameter in case of $|\epsilon|, |vk| \ll \tilde{\epsilon}_y$. This implies that the loop-cancellation is not suitable as a starting point to consider the low-energy excitation in two-dimension even for forward scattering model. Especially, the effect of the residual term is noticeable in the vicinity of $\epsilon = vk$.

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